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Quasi-Static Analysis of Three-Line Microstrip Symmetrical Coupler on Anisotropic Substrates

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***Abstract*—A method for analyzing the three symmetrically coupled microstrip lines on an anisotropic substrate has been developed. Computer programs based on the method of moments have been employed and the coupler mode impedance, Z, coupling constant, K, and phase velocity, v, as functions of the anisotropy ratio, $\epsilon_{xx}/\epsilon_{yy}$, have been obtained.**

I. Introduction

Three-line microstrip couplers are an integral part of many microwave communication systems, such as six-port reflectometers, balanced mixers, and phase shifters. In the analysis of three equal-width microstrip lines on isotropic substrates, it has been reported that under the assumption of a certain set of voltage modes, the characteristic impedance of the center line will always be the same as that of the side lines [1]. This imposes no condition on the capacitance coefficients of the network. Further work has revealed that for the same coupled system, the normal-mode impedances of the center line may be greater than those of side lines at small widths and separations; hence there are five normal-mode impedances [2]. It is necessary to increase the width of the center line relative to that of the outer lines in order to equalize the mode impedances. On the other hand, however, in the limit (W/h and $S/h > 0.8$), the two systems will be practically indistinguishable from each other.

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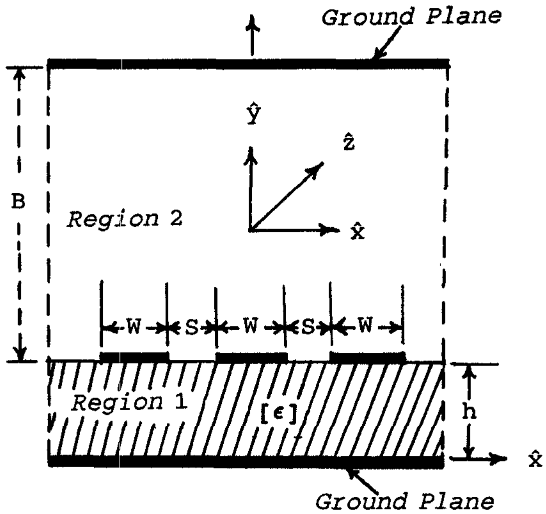

Fig. 1. Cross section of three coupled microstrip lines on an anisotropic substrate.

The boundary value problems involving single and coupled microstrip lines on anisotropic substrates such as sapphire and pyrolytic boron nitride (PBN) have been approached from numerical points of view [3]–[7]. Green's functions in the spectral domain have been utilized to transform an anisotropic problem to an isotropic one. It has been found for coupled lines that under the quasi-static TEM assumption, the difference between even- and odd-mode phase velocities can be significantly reduced by using anisotropic substrate. This implies that if an anisotropic material with a large anisotropy ratio is available, the isolation and directivity of microstrip couplers can be significantly improved.

In this paper, a system of three symmetrically coupled microstrip lines on an anisotropic substrate is analyzed. For a three-mode impedance system, the directivity of the three-line coupler is found to be improved by equalizing v_{eo} (square-root average of phase velocities of ee and oo modes) and v_{oe} (phase velocity of oe mode). The validity of our method has been verified by substituting the conditions of isotropic material in the equations derived for anisotropic material and then comparing the characteristic impedances with the results obtained in [1] and [2] for isotropic materials.

II. Derivation of Green's Function

The configuration of the three symmetrically coupled microstrip lines under consideration is shown in Fig. 1. It comprises three zero-thickness strips of width W and interstrip spacing S on an anisotropic dielectric substrate with thickness h. The introduction of ground cover by no means affects the solution as B is allowed to recede to infinity. In this derivation, an anisotropic substrate layer of homogeneous dielectric has been considered which has a relative permittivity tensor given by

$$\bar{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{bmatrix} \quad (1)$$

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and the elements of the matrix are expressed as

$$\epsilon_{11} = \epsilon_{xx}\cos^2\Theta + \epsilon_{yy}\sin^2\Theta$$
$$\epsilon_{12} = \epsilon_{21} = [\epsilon_{xx} - \epsilon_{yy}]\sin\Theta\cos\Theta$$
$$\epsilon_{22} = \epsilon_{xx}\sin^2\Theta + \epsilon_{yy}\cos^2\Theta \quad (2)$$

where the subscripts xx and yy refer to the crystal principal axes and Θ is the anisotropy angle.

A quasi-static solution to the potential problem can be obtained by solving Laplace's equation in the two dielectric regions:

$$\nabla(\bar{\epsilon}\cdot\nabla\phi_i) = 0, \qquad i = 1,2. \quad (3)$$

Owing to the infinite extent of the line in the z direction, the problem is two-dimensional. Therefore, (3) in the spectral domain yields

$$\left[\frac{\partial^2}{\partial y^2} + j\beta\frac{\epsilon_{12}+\epsilon_{21}}{\epsilon_{22}}\frac{\partial}{\partial y} - \beta^2\frac{\epsilon_{11}}{\epsilon_{22}}\right]\phi_i(\beta, y) = 0 \quad (4)$$

where β is the Fourier variable. The general solution of the Fourier-transformed potentials $\phi_i(\beta, y)$ in the spectral domain is

$$\phi_i(\beta, y) = e^{-j\beta R y}[A(\beta)\sinh(\beta T y) + B(\beta)\cosh(\beta T y)] \quad (5)$$

where

$$R = \frac{\epsilon_{12}+\epsilon_{21}}{2\epsilon_{22}} \qquad T = \left[\frac{\epsilon_{11}}{\epsilon_{22}} - \left(\frac{\epsilon_{12}+\epsilon_{21}}{2\epsilon_{22}}\right)^2\right]^{1/2}.$$

$A(\beta)$ and $B(\beta)$ are arbitrary coefficients that must be determined from the boundary conditions given by

$$\phi_1(\beta, 0) = 0$$
$$\phi_2(\beta, B) = 0$$
$$\phi_2(\beta, h+0) = \phi_1(\beta, h-0) = V(\beta)$$
$$\frac{\partial}{\partial y}\phi_2(\beta, h+0) = \epsilon_{22}\frac{\partial}{\partial y}\phi_1(\beta, h-0) - \frac{1}{\epsilon_0}\sigma(\beta) \quad (6)$$

where $V(\beta)$ and $\sigma(\beta)$ are the Fourier transforms of the potential on the air–dielectric interface and the charge density, respectively.

After applying the appropriate boundary conditions and setting $B \to \infty$, the transformed Green's function in the spectral domain is derived as

$$G(\beta, h) = V(\beta)/\sigma(\beta) = \frac{1}{\epsilon_0|\beta|[1 + \epsilon_{eq}\coth(|\beta| h_{eq})]} \quad (7)$$

and using inverse Fourier transformation with x_0 and x as source and field points, respectively, (7) can be written as

$$G(x, h) = \frac{1}{2\pi\epsilon_0(1+\epsilon_{eq})} \cdot \sum_{n=1}^{\infty}\rho^{n-1}\ln\frac{[(x-x_0)/h_{eq}]^2 + 4n^2}{[(x-x_0)/h_{eq}]^2 + 4(n-1)^2}. \quad (8)$$

Equation (8) can further be written as [8]

$$G(x, h) = \frac{1}{2\pi\epsilon_0}\left[a - b\ln\left|\frac{x-x_0}{2h_{eq}}\right| + c\sum_{n=1}^{\infty}\rho^n\ln\left[\left(\frac{x-x_0}{2nh_{eq}}\right)^2 + 1\right]\right] \quad (9)$$

where ϵ_{eq} and h_{eq} from [5] and a, b, c, and ρ from [8] are given as

$$\epsilon_{eq} = \epsilon_{22}\times T = \sqrt{\epsilon_{xx}\times\epsilon_{yy}} \qquad h_{eq} = T\times h = \frac{\alpha h}{(\alpha^2-1)\cos^2\Theta + 1}$$

$$\alpha = \sqrt{\epsilon_{yy}/\epsilon_{xx}} \qquad a = \frac{1-\rho^2}{\rho}\sum_{n=1}^{\infty}\rho^n\ln(n)$$

$$b = 1+\rho \qquad c = \frac{1-\rho^2}{2\rho} \quad \text{and} \quad \rho = \frac{1-\epsilon_{eq}}{1+\epsilon_{eq}}.$$

In the case of an isotropic material, $T = 1$ and $h_{eq} = h$.

III. Determination of Mode Characteristics

The static response to arbitrary excitation on triple-conductor microstrips is completely determined by the capacitance matrix [9]. The method of solution is as follows: The potentials of the center conductor and ground plane are assumed to be 1 and 0 V, respectively. Each conductor is divided into N subsections and the charge density is assumed to be uniform over each subsection, with an unknown charge density, σ_j, residing on the jth subsection. The boundary conditions at the ground plane and at the dielectric interface are satisfied by multiple imaging. The potential G_{ij} at the field point i caused by the unit charge on subsection j is written as

$$G_{ij} = \frac{1}{2\pi\epsilon_0}\left[a - b\ln\left|\frac{x_i - x_j}{2h_{eq}}\right| + c\sum_{n=1}^{\infty}\rho^n\ln\left[\left(\frac{x_i - x_j}{2\pi h_{eq}}\right)^2 + 1\right]\right]$$
$$\text{for } i, j = 1,2,3,\cdots, N \quad (10)$$

where

$$x_i - x_j = (p-q)\frac{S}{h} + (i - j - \xi + 0.5)\frac{W}{h}$$
$$p = q = 0,1,2$$
$$\xi = \begin{matrix} 1 & \text{for } |i - p\times N - 0.25\times N - 0.5| - (N-2)/4 \leqslant 0 \\ 0 & \text{for } |i - p\times N - 0.75\times N - 0.5| - (N-2)/4 \leqslant 0 \end{matrix}$$

and N is the number of subsections along the cross section of the center conductor.

Therefore, a matrix relationship is obtained between charge and voltage as

$$[\sigma] = [G]^{-1}[V]. \quad (11)$$

The matrix $[G]^{-1}$ is partitioned into submatrices, and the sum of the elements of each submatrix is identified as an element of the capacitance matrix, given as

$$[C] = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{12} \\ c_{13} & c_{12} & c_{11} \end{bmatrix}. \quad (12)$$

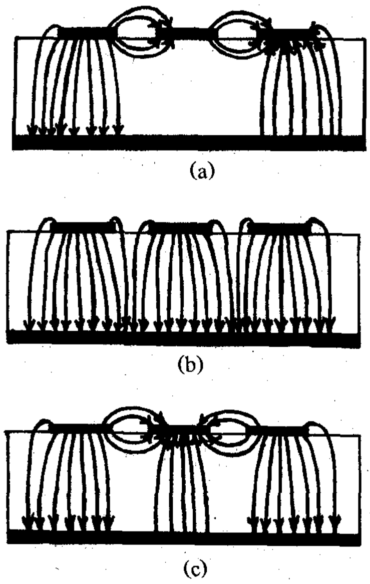

Fig. 2. Electric field lines for the fundamental modes in a three-line microstrip coupler: (a) *oe* mode; (b) *ee* mode; (c) *oo* mode.

For each of the three modes of propagation shown in Fig. 2, the capacitance for each of the three lines can be determined by using the method outlined in [2] and (12).

$$\text{for } oe \text{ mode:} \quad C_{oe} = c_{11} - c_{13}$$
$$\text{for } ee \text{ mode:} \quad C_{ee} = c_{11} + \mu c_{12} + c_{13}$$
$$\text{for } oo \text{ mode:} \quad C_{oo} = c_{11} + \frac{2}{\mu} c_{12} + c_{13}. \tag{13}$$

The parameter μ is an internal property of each particular network and is related to a certain set of voltage modes; it is given as

$$\mu = \frac{-(c_{11} - c_{22} + c_{13}) - \sqrt{(c_{11} - c_{22} + c_{13})^2 + 8c_{12}^2}}{2c_{12}}. \tag{14}$$

The corresponding mode impedances and phase velocities for each of the three modes are given by

$$Z_x = \frac{1}{c\sqrt{C_{x,\text{air}} C_x}} \tag{15}$$

$$v_x = c\sqrt{\frac{C_{x,\text{air}}}{C_x}} \tag{16}$$

where $x = oe$, ee, and oo for respective modes; c is the speed of light; $C_{x,\text{air}}$ is the static capacitance when the dielectric has been replaced by air; C_x is the capacitance with dielectric.

IV. Equalizing Phase Velocities to Improve Coupling and Isolation

The mode impedances, phase velocities, and coupling constants of three-line couplers are presented for anisotropic substrates. The microstrip conductors are assumed to be parallel to the $\hat{z}$ direction, separated by a distance S, and of equal width W, as shown in Fig. 1. By the Green's function method, the anisotropic problem is transformed into an isotropic one, and three independent propagating modes, namely *ee*, *oo*, and *oe*, are determined. The square-root average (Z_{eo}) of Z_{ee} and Z_{oo} is considered equivalent to even-mode impedance of the coupled lines; therefore, equalizing $v_{eo} = \sqrt{v_{ee} v_{oo}}$ and v_{oe} will improve the directivity of the three-line coupler [5].

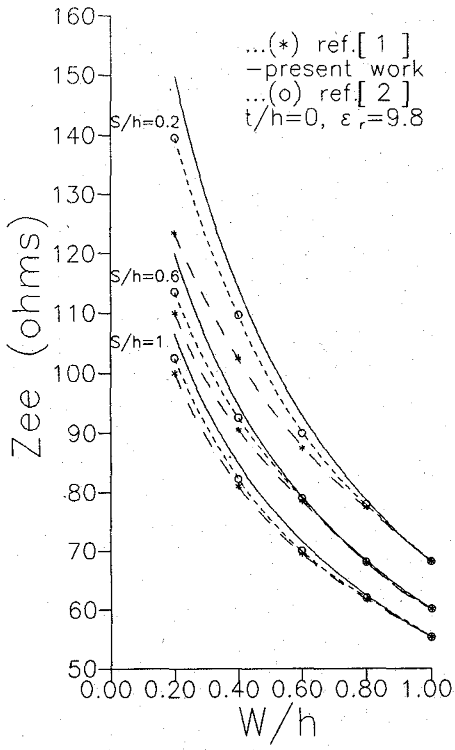

Fig. 3. Coupler mode impedance Z_{ee} for isotropic substrate for various W/h and S/h ratios.

The coupling constants of the coupler are defined as follows [3]:

$$K_2 = \frac{Z_{ee} - Z_{oo}}{Z_{ee} + Z_{oo}} \qquad K_{13} = \frac{\sqrt{Z_{ee} Z_{oo}} - Z_{oe}}{\sqrt{Z_{ee} Z_{oo}} + Z_{oe}} \tag{17}$$

where K_2 represents the coupling from the side lines into the center line, and K_{13} represents the coupling between the side lines through the center line. As the anisotropy ratio, $\epsilon_{xx}/\epsilon_{yy}$, increases, both K_2 and K_{13} increase, as expected. If the anisotropy ratio is large enough, usually $\geqslant 4$, the phase velocities can be equalized to improve the coupling and directivity. In order to equalize the phase velocities the anisotropy ratio, $\epsilon_{xx}/\epsilon_{yy}$, is varied with Θ and ϵ_{yy} is kept constant.

V. Numerical Results

The three-line coupler mode impedances Z_{oe}, Z_{ee}, and Z_{oo} for isotropic substrate have been computed after substituting the conditions of isotropic material in the equations derived for anisotropic material for $\epsilon_r = 9.8$ and various W/h and S/h ratios. These results are shown in Figs. 3, 4, and 5. For validity of our method these results have been compared with the results in [1] and [2]. A small difference, about 5%, is observed, especially in Figs. 3 and 5. The main reason for this difference is the number of subsections selected, which is 10 in the present case. The large number of subsections will approach the actual charge distribution on the strip, resulting in more accurate values, but it requires more computational time. Figs. 6, 7, and 8 show the coupling mode impedances, coupling constants, and phase velocities, respectively, as functions of the anisotropy ratio, $\epsilon_{xx}/\epsilon_{yy}$. The results are for $\Theta = 0$, $W/h = 0.8$, $S/h = 0.2$,

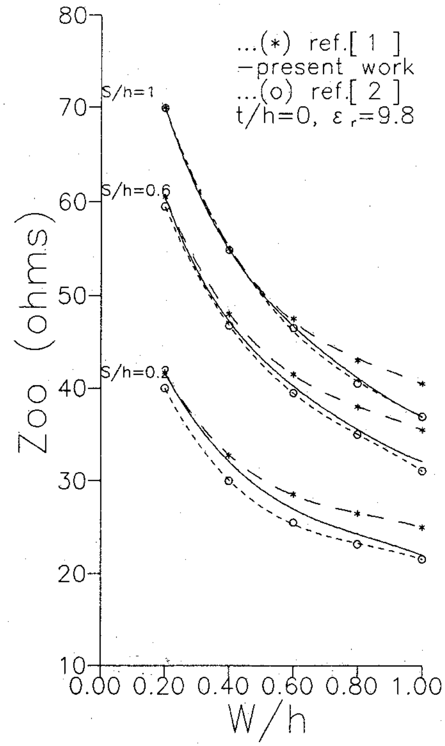

Fig. 4. Coupler mode impedance Z_{oo} for isotropic stubstrate for various W/h and S/h ratios.

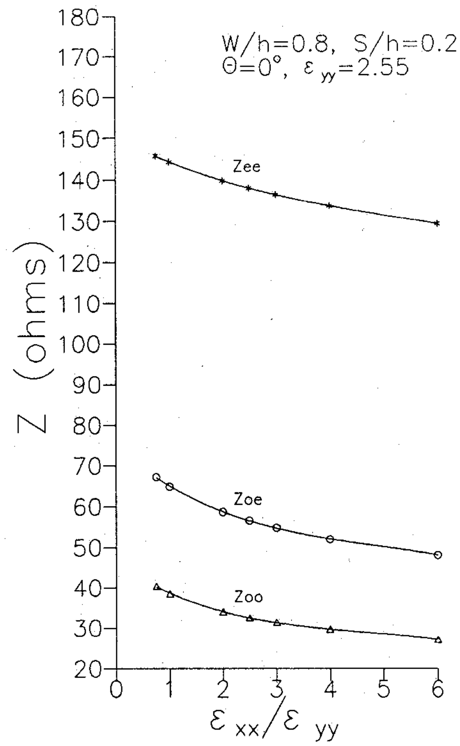

Fig. 6. Coupling mode impedances Z versus $\epsilon_{xx}/\epsilon_{yy}$.

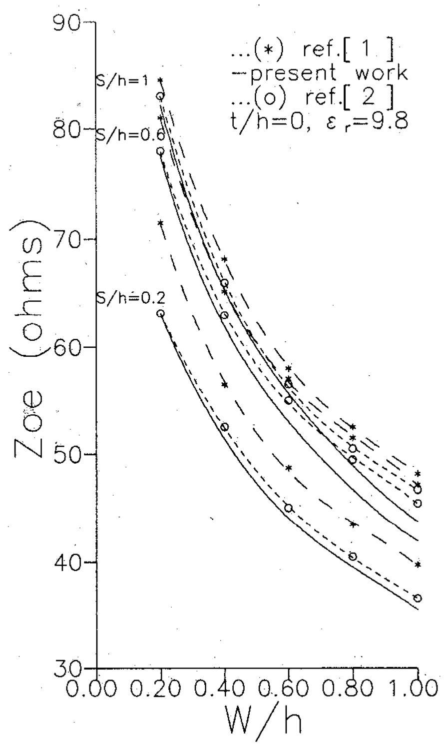

Fig. 5. Coupler mode impedance Z_{oe} for isotropic substrate for various W/h and S/h ratios.

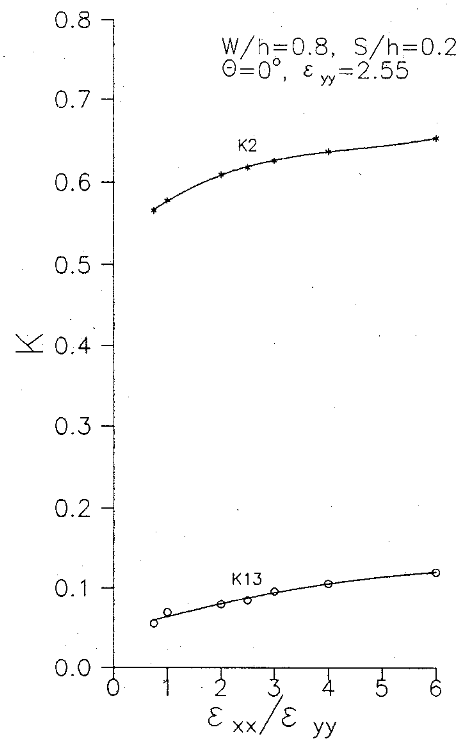

Fig. 7. Coupling constants versus $\epsilon_{xx}/\epsilon_{yy}$.

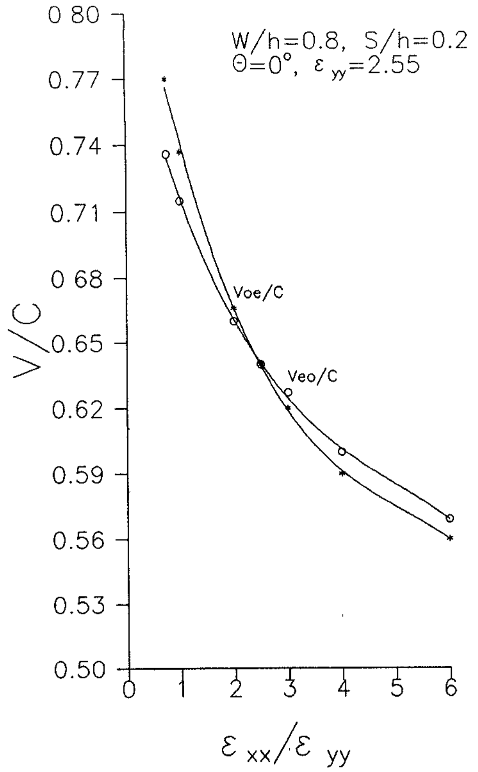

Fig. 8. Normalized phase velocities v/c versus $\epsilon_{xx}/\epsilon_{yy}$.

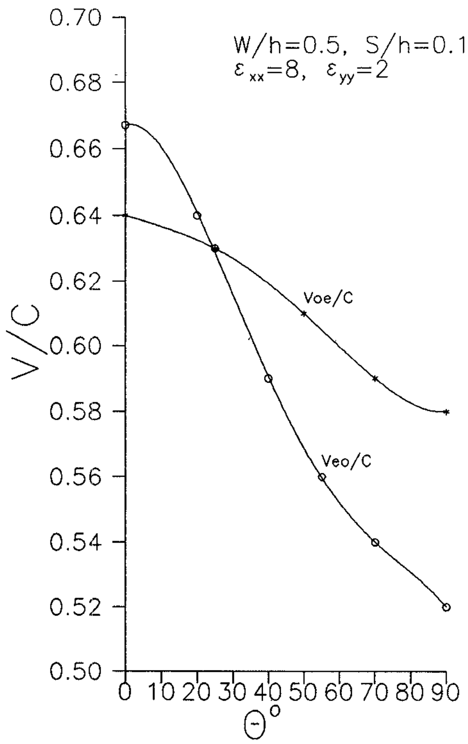

Fig. 9. Normalized phase velocities v/c versus anisotropic angle $\Theta°$.

and $\epsilon_{yy} = 2.55$ (polystyrenelike substrates). From Fig. 6, it is observed that the mode impedances decrease with $\epsilon_{xx}/\epsilon_{yy}$ because of stronger coupling between electric field lines. This results in an increased value of the capacitance and a decreased value of the impedance. The impedance for *ee* mode is quite large compared with *oe* and *oo* modes as a consequence of the almost zero coupling or lower capacitance value for this mode as the wave propagates through the strips without any coupling. Consequently, the coupling constants will increase with $\epsilon_{xx}/\epsilon_{yy}$, as shown in the Fig. 7. As expected, the normalized velocities decrease with increasing $\epsilon_{xx}/\epsilon_{yy}$ ratio, as shown in Fig. 8 and the velocity equalization takes place at $v/c = 0.646$ and $\epsilon_{xx}/\epsilon_{yy} = 2.4$ for $\Theta = 0$. In Fig. 9, it is shown that the phase velocity equalization for $\Theta \neq 0$ can take place only when an anisotropic substrate of $\epsilon_{xx}/\epsilon_{yy} \geqslant 4$ is used, thus proving the advantage of anisotropic materials with higher anisotropy for couplers. Normally, v_{oo} is always greater than v_{ee}. In an extreme case where $v_{ee} = v_{oo} = v_{eo}$, it can be proved that Z_{ee} will also be equal to Z_{oo}; hence the coupling constant K_2 will be equal to zero. This implies that there will be no coupling from the side lines to the center line, or that the same amount of energy coupled from line 1 into 2 will be immediately coupled into line 3. In this way, the coupler acts practically the same as a two-line coupler.

VI. Conclusion

The method discussed in this paper is a straightforward approach involving matrix inversion. The accuracy depends on the number of substrips, which requires more computational work. The effect of substrate anisotropy on the three-line microstrip coupler characteristics Z, v, and K has been demonstrated under the quasi-static TEM assumption. The use of anisotropic substrate is found to improve the coupler isolation and directivity by equalizing the coupler phase velocities. It also provides design flexibility by changing the characteristics of anisotropic materials, i.e., by varying ϵ_{xx}, ϵ_{yy}, and Θ values.

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